International Journal of Theoretical Physics, Vol. 44, No. 5, May 2005 (© 2005)
DOI: 10.1007/s10773-005-3985-6

Hamiltonian and Path Integral Formulations of the Born–Infeld Nambu–Goto $D1$-Brane Action with and Without a Dilation Field Under Gauge-Fixing

Usha Kulshreshtha[1] and D. S. Kulshreshtha[2,3]

Received September 1, 2004; accepted November 2, 2004

The Hamiltonian and path integral formulations of the Born–Infeld Nambu–Goto $D1$-brane action with and without a scalar dilation field are investigated under appropriate gauge-fixing.

KEY WORDS: Hamiltonian formulation; path integral formulation; Born–Infeld Nambu–Goto action; string theories; gauge theories.

1. INTRODUCTION

The Dirac–Born–Infeld Nambu–Goto (DBING) and the Born–Infeld Nambu–Goto (BING) actions are amongst the most important actions in the string theories (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, in press, 2003a,b; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). The Hamiltonian and path integral formulations of the first action has been studied by the present authors for the case of the $D1$-brane in (Kulshreshtha and Kulshreshtha, 2003a, 2004). The second action namely, the BING action is important in its own right for many reasons and has been studied from different points of view in the literature (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). In the present work, we study the Hamiltonian and path integral formulations (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976) of this BING action describing the $D1$-brane

[1] Department of Physics, Hindu College, University of Delhi, Delhi 110007, India.
[2] Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India.
[3] To whom correspondence should be addressed at Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India; e-mail: dsk@physics.du.ac.in; dsk@physik.uni-kl.de.

(sometimes also called the D-string) with and without a scalar dilation field φ under appropriate gauge-fixing conditions (GFC's).

In the next section, the action is considered without the dilation field and in Section 3, the action is studied in the presence of a scalar dilation field φ. The Hamiltonian and path integral quantizations are studied in both the cases under appropriate canonical gauge-fixing in the absence of boundary conditions (BC's). Finally the summary and discussion is presented in Section 4.

2. THE ACTION WITHOUT A DILATION FIELD

We consider the (bosonic) BING action describing the propagation of a $D1$-brane in a d-dimensional flat background (with $d = 10$ for the fermionic and $d = 26$ for the bosonic $D1$-brane) defined by (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, in press, 2003a,b; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996):

$$S_1 = \int \mathcal{L}_1 \, d^2\sigma \tag{1a}$$

$$\mathcal{L}_1 = (-T)[-\det(G_{\alpha\beta} + F_{\alpha\beta})]^{\frac{1}{2}} \tag{1b}$$

$$= (-T)[-\det(\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + F_{\alpha\beta})]^{\frac{1}{2}} \tag{1c}$$

$$= (-T)[-\det(\partial_\alpha X^\mu \partial_\beta X_\mu + (\partial_\alpha A_\beta - \partial_\beta A_\alpha))]^{\frac{1}{2}} \tag{1d}$$

$$= [-T][(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2 - f^2]^{\frac{1}{2}} \tag{1e}$$

$$G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}; \qquad F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \tag{1f}$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1, +1, \ldots + 1); \quad f = F_{01} = -F_{10} = (\dot{A}_1 - A_0') \tag{1g}$$

$$\mu, \nu = 0, 1, 2, \ldots, (d-1); \qquad \alpha, \beta = 0, 1 \tag{1h}$$

$$\dot{X}^\mu \equiv \frac{\partial X^\mu}{\partial \tau}; \quad X'^\mu = \frac{\partial X^\mu}{\partial \sigma}; \quad \dot{A}_1 \equiv \frac{\partial A_1}{\partial \tau}; \quad A_0' \equiv \frac{\partial A_0}{\partial \sigma} \tag{1i}$$

In the present work we would consider only the bosonic $D1$-brane with $d = 26$ (however, for the corresponding fermionic case one has $d = 10$). Here $\sigma^\alpha \equiv (\tau, \sigma)$ are the two parameters describing the world-sheet (WS). The overdots and primes denote in general, the derivatives with respect to the WS coordinates τ and σ. The string tension T is a constant of mass dimension two. $G_{\alpha\beta}$ is the induced metric on the WS and $X^\mu(\tau, \sigma)$ are the maps of the WS into the d-dimensional Minkowski space and describe the strings evolution in space–time (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, in press, 2003a,b; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). Here $F_{\alpha\beta}$ is the Maxwell field strength of the $U(1)$ gauge field $A_\alpha(\tau, \sigma)$. It is important to mention here that the $U(1)$ gauge field A_α is a scalar field in the target-space whereas it is an α - vector field in the WS-space. Also, we are considering the $U(1)$ gauge field A_α, to be a function only of the WS coordinates τ and σ and not of the target-space coordinates X^μ (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, in press, 2003a,b; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

Further the theory described by the action S_1 is a gauge-invariant (GI) (and consequently a gauge nonanomalous) theory possessing the usual three local gauge symmetries given by the two-dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, in press, 2003a,b; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). The canonical momenta obtained from $\mathcal{L}_1$ are

$$\Pi^\mu := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau X_\mu)} = [-T/L][(\dot{X}\cdot X')X'^\mu - (X')^2\dot{X}^\mu] \tag{2a}$$

$$\pi^0 := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau A_0)} = 0 \tag{2b}$$

$$E(\equiv \pi^1) := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau A_1)} = [T/L][f] \tag{2c}$$

$$L^2 = \left[(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2 - f^2\right] \tag{2d}$$

$$\partial_\tau \equiv \partial/\partial_\tau; \quad \partial_\sigma \equiv \partial/\partial_\sigma \tag{2e}$$

where Π^μ, π^0 and $E(\equiv \pi^1)$ are the canonical momenta conjugate respectively to X_μ, A_0, and A_1. The theory described by S_1 is thus seen to possess three primary constraints:

$$\psi_1 = \pi^0 \approx 0 \tag{3a}$$

$$\psi_2 = (\Pi\cdot X') \approx 0 \tag{3b}$$

$$\psi_3 = [\Pi^2 + (E^2 + T^2)(X')^2] \approx 0 \tag{3c}$$

Here the symbol $\approx$ denotes a weak equality (WE) in the sense of Dirac (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976), and it implies that these above constraints hold as strong equalities only on the reduced hypersurface of the constraints and not in the rest of the phase space of the classical theory (and similarly one can consider it as a weak operator equality (WOE) for the corresponding quantum theory) (Dirac, 1950).

The canonical Hamiltonian density corresponding to $\mathcal{L}_1$ is

$$\mathcal{H}_1^c = [\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) + E(\partial_\tau A_1) - \mathcal{L}_1] \tag{4a}$$

$$= [EA_0'] \tag{4b}$$

After incorporating the primary constraints of the theory in the canonical Hamiltonian density $\mathcal{H}_1^c$ with the help of Lagrange multiplier fields $u_1(\tau,\sigma)$, $u_2(\tau,\sigma)$, and $u_3(\tau,\sigma)$, which we treat as dynamical, the total Hamiltonian density of the theory could be written as

$$\mathcal{H}_1^T = \left[\mathcal{H}_1^c + u_1\psi_1 + u_2\psi_2 + u_3\psi_3\right] \tag{5a}$$

$$= [EA_0' + u_1\pi^0 + u_2(\Pi\cdot X') + u_3[\Pi^2 + (E^2+T^2)(X')^2]] \tag{5b}$$

We treat u_1, u_2, and u_3 as dynamical.Also the momenta conjugate to u_1, u_2, and u_3 are denoted respectively by p_{u_1}, p_{u_2}, and p_{u_3}. The Hamiltons equations of motion obtained from the total Hamiltonian

$$H_1^T = \int \mathcal{H}_1^T d\sigma \tag{6}$$

e.g., for the closed strings with the periodic BC's are

$$+\partial_\tau X^\mu = \frac{\partial H_1^T}{\partial \Pi_\mu} = [u_2 X'^\mu + 2\Pi^\mu u_3] \tag{7a}$$

$$-\partial_\tau \Pi^\mu = \frac{\partial H_1^T}{\partial X_\mu} = -\partial_\sigma[u_2\Pi^\mu + 2X'^\mu(E^2+T^2)u_3] \tag{7b}$$

$$+\partial_\tau A_0 = \frac{\partial H_1^T}{\partial \pi^0} = u_1 \tag{7c}$$

$$-\partial_\tau \pi^0 = \frac{\partial H_1^T}{\partial A_0} = [-E'] \tag{7d}$$

$$+\partial_\tau A_1 = \frac{\partial H_1^T}{\partial E} = [A_0' + 2E(X')^2 u_3] \tag{7e}$$

$$-\partial_\tau E = \frac{\partial H_1^T}{\partial A_1} = 0 \tag{7f}$$

$$+\partial_\tau u_1 = \frac{\partial H_1^T}{\partial p_{u_1}} = 0 \tag{7g}$$

$$-\partial_\tau p_{u_1} = \frac{\partial H_1^T}{\partial u_1} = \pi^0 \tag{7h}$$

$$+\partial_\tau u_2 = \frac{\partial H_1^T}{\partial p_{u_2}} = 0 \tag{7i}$$

$$-\partial_\tau p_{u_2} = \frac{\partial H_1^T}{\partial u_2} = (\Pi \cdot X') \tag{7j}$$

$$+\partial_\tau u_3 = \frac{\partial H_1^T}{\partial p_{u_3}} = 0 \tag{7k}$$

$$-\partial_\tau p_{u_3} = \frac{\partial H_1^T}{\partial u_3} = [\Pi^2 + (E^2 + T^2)(X')^2] \tag{7l}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraint ψ_1 be preserved in the course of time one obtains a secondary constraint (with a Poisson bracket (PB) being denoted by $\{\ \ ,\ \ \}_p$):

$$\psi_4 = \left\{\psi_1, \mathcal{H}_1^T\right\}_p = [E^{'}] \approx 0 \tag{8}$$

The preservation of ψ_4 for all time does not give rise to any further constraints. Similarly, the preservation of ψ_2 and ψ_3 for all time also does not yield any further constraints. The theory is thus seen to possess only four constraints ψ_1, ψ_2, ψ_3, and ψ_4. Also the first-order Lagrangian density of the theory is

$$\begin{aligned}\mathcal{L}_1^{IO} &= \left[\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) + E(\partial_\tau A_1) + p_{u_1}(\partial_\tau u_1)\right.\\ &\qquad \left. + p_{u_2}(\partial_\tau u_2) + p_{u_3}(\partial_\tau u_3) - \mathcal{H}_1^T\right] &(9a)\\ &= [\Pi^2 + (E^2 - T^2)(X')^2]u_3 &(9b)\end{aligned}$$

The matrix of the Poisson brackets of the constraints ψ_i is seen to be a singular matrix implying that the set of constraints ψ_i is first-class (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976) and that the theory described by S_1 is a gauge-invariant (GI) theory. It is rather well known that the theory described by S_1 indeed possesses three local gauge symmetries given by the two-dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, in press, 2003a,b; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

To study the Hamiltonian and path integral formulations of this theory under gauge-fixing, we convert the set of first-class constraints of the theory ψ_i into a set of second-class constraints, by imposing arbitrarily, some additional constraints on the system called the gauge-fixing conditions (GFC's) or the gauge constraints. For this purpose, we could choose, for example, the set of GFC's (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976):

$$\psi_5 = \zeta_1 = X^2 \approx 0 \tag{10a}$$

$$\psi_6 = \zeta_2 = \Pi' \approx 0 \tag{10b}$$

$$\psi_7 = \zeta_3 = A_1 \approx 0 \tag{10c}$$

$$\psi_8 = \zeta_4 = A_0 \approx 0 \tag{10d}$$

Corresponding to this choice of GFC's, the total set of constraints of the theory under which the quantization of the theory could, e.g., be studied becomes

$$\psi_1 = \pi^0 \approx 0 \tag{11a}$$

$$\psi_2 = (\Pi \cdot X') \approx 0 \tag{11b}$$

$$\psi_3 = [\Pi^2 + (E^2 + T^2)(X')^2] \approx 0 \tag{11c}$$

$$\psi_4 = E' \approx 0 \tag{11d}$$

$$\psi_5 = \zeta_1 = X^2 \approx 0 \tag{11e}$$

$$\psi_6 = \zeta_2 = \Pi' \approx 0 \tag{11f}$$

$$\psi_7 = \zeta_3 = A_1 \approx 0 \tag{11g}$$

$$\psi_8 = \zeta_4 = A_0 \approx 0 \tag{11h}$$

We now calculate the matrix $M_{\alpha\beta}(:= \{\psi_\alpha, \psi_\beta\}_p)$ of the Poisson brackets of the constraints ψ_i. The nonvanishing elements of the matrix $M_{\alpha\beta}$ are obtained as:

$$M_{18} = -M_{81} = [-1]\,\delta\,(\sigma - \sigma') \tag{12a}$$

$$M_{25} = -M_{52} = [-2X']\,\delta\,(\sigma - \sigma') \tag{12b}$$

$$M_{26} = -M_{62} = [-\Pi]\,\delta''\,(\sigma - \sigma') \tag{12c}$$

$$M_{35} = -M_{53} = [-4\Pi]\,\delta\,(\sigma - \sigma') \tag{12d}$$

$$M_{36} = -M_{63} = [-2X'(E^2 + T^2)]\,\delta''\,(\sigma - \sigma') \tag{12e}$$

$$M_{37} = -M_{73} = [-2E(X')^2]\,\delta\,(\sigma - \sigma') \tag{12f}$$

$$M_{47} = -M_{74} = [-1]\,\delta'\,(\sigma - \sigma') \tag{12g}$$

The matrix $M_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding *set* of constraints ψ_i is a set of second-class constraints (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976). The determinant of the matrix

$M_{\alpha\beta}$ is given by

$$[\| \det(M_{\alpha\beta})\|]^{\frac{1}{2}} = [4M\delta''(\sigma - \sigma')\,\delta'(\sigma - \sigma')\,\delta^2(\sigma - \sigma')] \tag{13a}$$

$$M = [\Pi^2 - (E^2 + T^2)(X')^2] \tag{13b}$$

The nonvanishing elements of the inverse of the matrix $M_{\alpha\beta}$ (i.e., the elements of the matrix $(M^{-1})_{\alpha\beta}$) are

$$(M^{-1})_{18} = -(M^{-1})_{81} = \delta\,(\sigma - \sigma') \tag{14a}$$

$$(M^{-1})_{25} = -(M^{-1})_{52} = [-(E^2 + T^2)X'/(2M)]\,\delta\,(\sigma - \sigma') \tag{14b}$$

$$(M^{-1})_{26} = -(M^{-1})_{62} = [\Pi/(2M)]|\sigma - \sigma'| \tag{14c}$$

$$(M^{-1})_{35} = -(M^{-1})_{53} = [\Pi/(4M)]\delta\,(\sigma - \sigma') \tag{14d}$$

$$(M^{-1})_{36} = -(M^{-1})_{63} = [-(X')/(4M)]|\sigma - \sigma'| \tag{14e}$$

$$(M^{-1})_{45} = +(M^{-1})_{54} = [\Pi E(X')^2/(4M)]\,\epsilon\,(\sigma - \sigma') \tag{14f}$$

$$(M^{-1})_{46} = +(M^{-1})_{64} = [-E(X')^2(X^{'})/(4M)]$$
$$|\sigma - \sigma'|\,\epsilon\,(\sigma - \sigma')\,\delta\,(\sigma - \sigma') \tag{14g}$$

$$(M^{-1})_{47} = +(M^{-1})_{74} = (-1/2)\,\epsilon\,(\sigma - \sigma') \tag{14h}$$

with the step functions $\epsilon(\sigma - \sigma')$ defined as

$$\epsilon(\sigma - \sigma') := \begin{cases} +1, \ (\sigma - \sigma') > 0 \\ -1, \ (\sigma - \sigma') < 0 \end{cases} \tag{15}$$

and

$$\int M(\sigma, \sigma'')M^{-1}(\sigma'', \sigma')d\sigma'' = \mathbf{1}_{8\times 8}\delta(\sigma - \sigma') \tag{16}$$

Now following the standard Dirac quantization procedure in the Hamiltonian formulation (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 2004, 2002a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976), the nonvanishing equal WS time (EWST) Dirac brackets of the theory described by the action S_1 under the GFC's ζ_i could be obtained easily after a lengthy but straightforward calculation (Dirac, 1950) and are omitted here for the sake of brevity.

It is important to recall here that the constraints of the theory represent only the weak equalities in the sense of Dirac (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976), as explained in the foregoing implying that they are strongly zero only on the reduced hypersurface of the constraints and not in the rest of the phase space of the (classical) theory (with a similar weak operator equality holding for the corresponding quantum theory) and as a consequence of this the DB's involving the gauge fields like A_α can indeed be nonvanishing in principle (as is evident in the present case from the above results) which would , however, become strongly zero on the reduced hypersurface of the constraints of the theory described by the action in any case.

Further, in the canonical quantization of the theory while going from equal WS time (EWST) Dirac brackets of the theory to the corresponding EWST commutation relations one would encounter here the problem of operator ordering (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Maharana, 1983; Senjanovic, 1976) because the product of canonical variables of the theory are involved in the classical description of the theory (like in the expressions for the constraints of the theory) as well as in the calculation of the Dirac brackets. These variables are envisaged as noncommuting operators in the quantized theory leading to the problem of so-called operator ordering (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Maharana, 1983; Senjanovic, 1976). This problem could, however, be resolved (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Maharana, 1983; Senjanovic, 1976) by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the Hermiticity of these operators (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Maharana, 1983; Senjanovic, 1976).

In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z_1[J_i]$ of the theory under GFC's ζ_i in the presence of the external sources J_i (following the Senjanovic procedure (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Senjanovic, 1976) for a theory possessing a set of second-class constraints, appropriate for our theory described by the action S_1 considered under the GFC's: ζ_i(12) Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Senjanovic, 1976)) as follows:

$$Z_1[J_i] = \int [d\mu] \exp \Bigg[i \int d^2\sigma \big[J_i \Phi^i + \Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) \tag{17a}$$

$$+ E(\partial_\tau A_1) + p_{u_1}(\partial_\tau u_1) + p_{u_2}(\partial_\tau u_2) + p_{u_3}(\partial_\tau u_3) - \mathcal{H}_1^T \big] \Bigg] \tag{17b}$$

where the phase space variables of the theory are $\Phi^i \equiv (X^\mu, A_0, A_1, u_1, u_2, u_3)$ with the corresponding respective canonical conjugate momenta:

$\Pi_i \equiv (\Pi_\mu, \pi^0, E, p_{u_1}, p_{u_2}, p_{u_3})$. The functional measure $[d\mu]$ of the generating functional $Z_1[J_i]$ under the GFC's ζ_i is obtained using Eqs. (9), (11) and (13) as

$$\begin{aligned}[d\mu] = {} & [4M\delta''(\sigma-\sigma')\delta'(\sigma-\sigma')\delta^2(\sigma-\sigma')][dX^\mu][dA_0][dA_1][du_1][du_2][du_3] \\ & \times [d\Pi_\mu][d\pi^0][dE][dp_{u_1}][dp_{u_2}][dp_{u_3}] \cdot \delta[(\pi^0)\approx 0]\cdot\delta[(\Pi\cdot X') \\ & \approx 0]\cdot\delta[[\Pi^2+(E^2+T^2)(X')^2]\approx 0]\cdot\delta[(E')\approx 0]\cdot\delta[(X^2) \\ & \approx 0]\cdot\delta[(\Pi')\approx 0]\cdot\delta[(A_1)\approx 0]\cdot\delta[(A_0)\approx 0] \end{aligned} \tag{18}$$

The Hamiltonian and path integral quantization of the theory described by the action S_1 under the GFC's ζ_i is now complete. In the next section we study this theory in the presence of the scalar dilation field.

3. THE ACTION IN THE PRESENCE OF A SCALAR DILATION FIELD

The (bosonic) BING action describing the propagation of a $D1$-brane in a d-dimensional flat background in the presence of a scalar dilation field φ is defined by (Abou and Hull, 1997; Aganagic *et al*., 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b, in press; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996):

$$S_2 = \int \mathcal{L}_2 d^2\sigma \tag{19a}$$

$$\mathcal{L}_2 = [e^{-\varphi}\mathcal{L}_1] \tag{19b}$$

$$= [-Te^{-\varphi}][(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2 - f^2]^{\frac{1}{2}} \tag{19c}$$

$$= [-Te^{-\varphi}]L \tag{19d}$$

The canonical momenta obtained from $\mathcal{L}_2$ are

$$\Pi^\mu := \frac{\partial\mathcal{L}_2}{\partial(\partial_\tau X_\mu)} = [-Te^{-\varphi}/L][(\dot{X}\cdot X')X'^\mu - (X')^2\dot{X}^\mu] \tag{20a}$$

$$\pi^0 := \frac{\partial\mathcal{L}_2}{\partial(\partial_\tau A_0)} = 0 \tag{20b}$$

$$E(\equiv \pi^1) := \frac{\partial\mathcal{L}_2}{\partial(\partial_\tau A_1)} = [Te^{-\varphi}/L]f \tag{20c}$$

$$\pi := \frac{\partial\mathcal{L}_2}{\partial(\partial_\tau \varphi)} = 0 \tag{20d}$$

Here π is the momentum canonically conjugate to the dilation field φ. The theory described by S_2 is thus seen to possess four primary constraints:

$$\chi_1 = \pi \approx 0 \tag{21a}$$

$$\chi_2 = \pi^0 \approx 0 \tag{21b}$$

$$\chi_3 = (\Pi \cdot X') \approx 0 \tag{21c}$$

$$\chi_4 = [\Pi^2 + (E^2 + T^2 e^{-2\varphi})(X')^2] \approx 0 \tag{21d}$$

The canonical Hamiltonian density corresponding to $\mathcal{L}_2$ is

$$\mathcal{H}_2^c = \left[\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) + E(\partial_\tau A_1) + \pi(\partial_\tau \varphi) - \mathcal{L}_2\right] \tag{22a}$$

$$= [E A_0'] \tag{22b}$$

After incorporating the primary constraints of the theory in the canonical Hamiltonian density of the theory $\mathcal{H}_2^c$ with the help of Lagrange multiplier fields $v_1(\tau, \sigma)$, $v_2(\tau, \sigma)$, $v_3(\tau, \sigma)$, and $v_4(\tau, \sigma)$, which we treat as dynamical, the total Hamiltonian density of the theory could be written as

$$\mathcal{H}_2^T = [\mathcal{H}_2^c + v_1\chi_1 + v_2\chi_2 + v_3\chi_3 + v_4\chi_4] \tag{23a}$$

$$\begin{aligned} &= [E A_0' + v_1\pi + v_2\pi^0 + v_3(\Pi \cdot X') \\ &\quad + v_4[\Pi^2 + (E^2 + T^2 e^{-2\varphi})(X')^2]] \end{aligned} \tag{23b}$$

The momenta canonically conjugate to v_1, v_2, v_3, and v_4 will be denoted respectively by p_{v_1}, p_{v_2}, p_{v_3}, and p_{v_4}. The Hamiltons equation of motion obtained from the total Hamiltonian:

$$H_2^T = \int \mathcal{H}_2^T d\sigma \tag{24}$$

e.g. for the closed strings with periodic BC's are

$$+\partial_\tau X^\mu = \frac{\partial H_2^T}{\partial \Pi_\mu} = [v_3 X'^\mu + 2\Pi^\mu v_4] \tag{25a}$$

$$-\partial_\tau \Pi^\mu = \frac{\partial H_2^T}{\partial X_\mu} = -\partial_\sigma[v_3\Pi^\mu + 2X'^\mu(E^2 + T^2 e^{-2\varphi})v_4] \tag{25b}$$

$$+\partial_\tau A_0 = \frac{\partial H_2^T}{\partial \pi^0} = v_2 \tag{25c}$$

$$-\partial_\tau \pi^0 = \frac{\partial H_2^T}{\partial A_0} = [-E'] \tag{25d}$$

$$+\partial_\tau A_1 = \frac{\partial H_2^T}{\partial E} = [A_0' + 2E(X')^2 v_4] \tag{25e}$$

$$-\partial_\tau E = \frac{\partial H_2^T}{\partial A_1} = 0 \tag{25f}$$

$$+\partial_\tau \varphi = \frac{\partial H_2^T}{\partial \pi} = v_1 \tag{25g}$$

$$-\partial_\tau \pi = \frac{\partial H_2^T}{\partial \varphi} = [-2T^2 e^{-2\varphi}(X')^2 v_4] \tag{25h}$$

$$+\partial_\tau v_1 = \frac{\partial H_2^T}{\partial p_{v_1}} = 0 \tag{25i}$$

$$-\partial_\tau p_{v_1} = \frac{\partial H_2^T}{\partial v_1} = \pi \tag{25j}$$

$$+\partial_\tau v_2 = \frac{\partial H_2^T}{\partial p_{v_2}} = 0 \tag{25k}$$

$$-\partial_\tau p_{v_2} = \frac{\partial H_2^T}{\partial v_2} = \pi^0 \tag{25l}$$

$$+\partial_\tau v_3 = \frac{\partial H_2^T}{\partial p_{v_3}} = 0 \tag{25m}$$

$$-\partial_\tau p_{v_3} = \frac{\partial H_2^T}{\partial v_3} = [\Pi \cdot X'] \tag{25n}$$

$$+\partial_\tau v_4 = \frac{\partial H_2^T}{\partial p_{v_4}} = 0 \tag{25o}$$

$$-\partial_\tau p_{v_4} = \frac{\partial H_2^T}{\partial v_4} = [\Pi^2 + (E^2 + T^2 e^{-2\varphi})(X')^2] \tag{25p}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraint χ_2 be preserved in the course of time one obtains a secondary constraint

$$\chi_5 = \left\{\chi_2, \mathcal{H}_2^T\right\}_P = [E'] \approx 0 \tag{26}$$

The presentation of χ_5 for all time gives rise to another secondary constraint. Similarly the preservation of χ_2, χ_3, and χ_4 for all time does not yield any further constraints. The theory is thus seen to possess only five constraints χ_1, χ_2, χ_3, χ_4, and χ_5. Also the first-order Lagrangian density of the theory (to be used later) is

$$\begin{aligned}\mathcal{L}_2^{IO} &= \big[\Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) + E(\partial_\tau A_1) + \pi(\partial_\tau \varphi) + p_{v_1}(\partial_\tau v_1) \\ &\quad + p_{v_2}(\partial_\tau v_2) + p_{v_3}(\partial_\tau v_3) + p_{v_4}(\partial_\tau v_4) - \mathcal{H}_2^T\big] \qquad (27a) \\ &= [\Pi^2 + (E^2 - T^2 e^{-2\varphi})(X')^2] v_4 \qquad (27b)\end{aligned}$$

The matrix of the Poisson brackets of the constraints χ_i is seen to be a singular matrix implying that the set of constraints χ_i is first-class (Kulshreshtha and Kulshreshtha, 2003a,b, 2004; Gitman and Tyutin, 1990; Senjanovic, 1976; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b; Kulshreshtha and Kulshreshtha, 1993c; Kulshreshtha *et al*., 1993d,e; Dirac, 1950) and that the theory described by S_2 is a gauge-invariant (GI) theory. It is in fact, well known to posses three local gauge symmetries given by the two-dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) (Abou and Hull, 1997; Aganagic *et al*., 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b, in press; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

To study the Hamiltonian and path integral formulations of this GI theory under GFC's, we convert the set of first-class constraints of the theory χ_i into a set of second-class constraints, by imposing arbitrarily, some additional constraints on the system called the GFC's or the gauge constraints. For this purpose, we could choose, for example, the set of GFC's (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al*., 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976):

$$\chi_6 = \rho_1 = X^2 \approx 0 \tag{28a}$$

$$\chi_7 = \rho_2 = \Pi' \approx 0 \tag{28b}$$

$$\chi_8 = \rho_3 = A_1 \approx 0 \tag{28c}$$

$$\chi_9 = \rho_4 = A_0 \approx 0 \tag{28d}$$

$$\chi_{10} = \rho_5 = \varphi \approx 0 \tag{28e}$$

Corresponding to this choice of GFC's, the total set of constraints of the theory under which the quantization of the theory could, e.g., be studied becomes

$$\chi_1 = \pi \approx 0 \tag{29a}$$

$$\chi_2 = \pi^0 \approx 0 \tag{29b}$$

$$\chi_3 = (\Pi \cdot X') \approx 0 \tag{29c}$$

$$\chi_4 = [\Pi^2 + (E^2 + T^2 e^{-2\varphi}(X')^2] \approx 0 \tag{29d}$$

$$\chi_5 = E' \approx 0 \tag{29e}$$

$$\chi_6 = \rho_1 = X^2 \approx 0 \tag{29f}$$

$$\chi_7 = \rho_2 = \Pi' \approx 0 \tag{29g}$$

$$\chi_8 = \rho_3 = A_1 \approx 0 \tag{29h}$$

$$\chi_9 = \rho_4 = A_0 \approx 0 \tag{29i}$$

$$\chi_{10} = \rho_5 = \varphi \approx 0 \tag{29j}$$

We now calculate the matrix $R_{\alpha\beta}(:= \{\chi_\alpha, \chi_\beta\}_P)$ of the Poisson brackets of the constraints χ_i. The nonvanishing elements of the matrix $R_{\alpha\beta}$ are obtained as

$$R_{14} = -R_{41} = [2T^2 e^{-2\varphi}(X')^2]\,\delta\,(\sigma - \sigma') \tag{30a}$$

$$R_{1,10} = -R_{10,1} = [-1]\,\delta(\sigma - \sigma') \tag{30b}$$

$$R_{29} = -R_{92} = [-1]\,\delta(\sigma - \sigma') \tag{30c}$$

$$R_{36} = -R_{63} = [-2X']\,\delta(\sigma - \sigma') \tag{30d}$$

$$R_{37} = -R_{73} = [-\Pi]\,\delta''\,(\sigma - \sigma') \tag{30e}$$

$$R_{46} = -R_{64} = [-4\Pi]\,\delta(\sigma - \sigma') \tag{30f}$$

$$R_{47} = -R_{74} = [-2(X')(E^2 + T^2 e^{-2\varphi})]\,\delta''\,(\sigma - \sigma') \tag{30g}$$

$$R_{48} = -R_{84} = [-2E(X')^2]\,\delta(\sigma - \sigma') \tag{30h}$$

$$R_{58} = -R_{85} = [-1]\,\delta(\sigma - \sigma') \tag{30i}$$

The matrix $R_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of constraints χ_i is a set of second-class constraints (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976). The determinant of the matrix $R_{\alpha\beta}$ is given by:

$$[\|\det(R_{\alpha\beta})\|]^{\frac{1}{2}} = [4R\delta''(\sigma - \sigma')\,\delta'\,(\sigma - \sigma')\,\delta^3(\sigma - \sigma')] \tag{31a}$$

$$R = [\Pi^2 - (E^2 + T^2 e^{-2\varphi})(X')^2] \tag{31b}$$

The nonvanishing elements of the inverse of the matrix $R_{\alpha\beta}$ (i.e., the elements of the matrix $(R^{-1})_{\alpha\beta}$) are:

$$(R^{-1})_{1,10} = -(R^{-1})_{10,1} = \delta\,(\sigma - \sigma') \tag{32a}$$

$$(R^{-1})_{29} = -(R^{-1})_{92} = \delta\,(\sigma - \sigma') \tag{32b}$$

$$(R^{-1})_{36} = -(R^{-1})_{63} = [-(X')(E^2 + T^2 e^{-2\varphi})/(2R)]\delta\,(\sigma - \sigma') \tag{32c}$$

$$(R^{-1})_{37} = -(R^{-1})_{73} = [\Pi/(2R)]|\sigma - \sigma'| \tag{32d}$$

$$(R^{-1})_{46} = -(R^{-1})_{64} = [\Pi/(4R)]\,\delta\,(\sigma - \sigma') \tag{32e}$$

$$(R^{-1})_{47} = -(R^{-1})_{74} = [-(X')/(4R)]|\sigma - \sigma'| \tag{32f}$$

$$(R^{-1})_{56} = (R^{-1})_{65} = [\Pi E(X')^2/(4R)]\,\epsilon\,(\sigma - \sigma') \tag{32g}$$

$$\begin{aligned}(R^{-1})_{57} = (R^{-1})_{75} &= [-E(X')(X')^2/(4R)]|\sigma - \sigma'| \\ &\quad \times \epsilon\,(\sigma - \sigma')\,\delta\,(\sigma - \sigma')\end{aligned} \tag{32h}$$

$$(R^{-1})_{58} = (R^{-1})_{85} = (-1/2)\epsilon\,(\sigma - \sigma') \tag{32i}$$

$$(R^{-1})_{6,10} = -(R^{-1})_{10,6} = [-\Pi(X')^2(T^2e^{-2\varphi})/(2R)]\,\delta\,(\sigma - \sigma') \tag{32j}$$

with

$$\int R(\sigma, \sigma'')R^{-1}(\sigma'', \sigma')d\sigma'' = \mathbf{1}_{10\times 10}\,\delta(\sigma - \sigma') \tag{33}$$

Now following the standard Dirac quantization procedure in the Hamiltonian formulation (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 2004, 2002a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976), the nonvanishing EWST Dirac brackets of the theory in the presence of a scalar dilation field described by the action S_2 under the GFC's ρ_i could again be obtained after a lengthy but straightforward calculation (Dirac, 1950) and are omitted here again for the sake of brevity.

As explained in the previous section, the nonvanishing DB's involving the gauge field A_1, in the above results, would become strongly zero on the reduced hypersurface of the constraints of the theory described by the action S_2 (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Senjanovic, 1976).

The problem of operator ordering occurring here while making a transition from the EWST Dirac brackets to the corresponding EWST commutation relations can be resolved here as explained in Section 3, by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Maharana, 1983; Senjanovic, 1976).

In the path integral formulation, the transition to quantum theory is made again by writing the vacuum to vacuum transition amplitude for the theory, called the generating functional $Z_2[J_i]$ of the theory, following again the Senjanovic procedure for a theory possessing a set of second-class constraints (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, in press, 2002a,b, 2003a,b; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Senjanovic, 1976), appropriate for our theory described by the action S_2 considered under the GFC's ρ_i, in the presence of the external sources J_i as follows (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b,1994b, 1993d,e; Senjanovic, 1976):

$$Z_2[J_i] = \int [d\mu] \exp[i \int d^2\sigma \big[J_i\Phi^i + \Pi^\mu(\partial_\tau X_\mu) + \pi^0(\partial_\tau A_0) \tag{34a}$$

$$+ E(\partial_\tau A_1) + \pi(\partial_\tau \varphi) + p_{v_1}(\partial_\tau v_1) + p_{v_2}(\partial_\tau v_2) + p_{v_3}(\partial_\tau v_3)$$

$$+ p_{v_4}(\partial_\tau v_4) - \mathcal{H}_2^T\big]] \tag{34b}$$

where the phase space variables of the theory are $\Phi^i \equiv (X^\mu, A_0, A_1, \varphi, v_1, v_2, v_3, v_4)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_\mu, \pi^0, E, \pi, p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4})$. The functional measure $[d\mu]$ of the generating functional $Z_2[J_i]$ under the GFC's ρ_i is obtained using Eqs. (27), (29), and (31) as

$$
\begin{aligned}
[d\mu] = {} & [4M\delta''(\sigma - \sigma')\delta'(\sigma - \sigma')\delta^3(\sigma - \sigma')] \\
& \times [dX^\mu][dA_0][dA_1][d\varphi][dv_1][dv_2][dv_3][dv_4] \\
& \times [d\Pi_\mu][d\pi^0][dE][d\pi][dp_{v_1}][dp_{v_2}][dp_{v_3}][dp_{v_4}] \cdot \delta[(\pi) \\
& \approx 0] \cdot \delta[(\pi^0) \approx 0]\delta(\Pi \cdot X') \approx 0] \cdot [[\Pi^2 + (E^2 \\
& + T^2 e^{-2*\varphi})(X')^2] \approx 0] \cdot \delta[(E') \approx 0] \cdot \delta[(X^2) \approx 0] \cdot \delta[(\Pi') \\
& \approx 0] \cdot \delta[(A_1) \approx 0] \cdot \delta[(A_0) \approx 0] \cdot \delta[(\varphi) \approx 0]
\end{aligned} \tag{35}
$$

The Hamiltonian and path integral quantization of the theory described by the action S_2 under the GFC's ρ_i is now complete.

4. SUMMARY AND DISCUSSION

In this work we have studied the Hamiltonian and path integral quantization of the BING action describing the $D1$-brane action with and without a scalar dilation field φ under appropriate GFC·s in the absence of BC‘s, using the instant-form of dynamics on the hyperplanes of the WS defined by the hyperplanes: WS-time $= \sigma^0 = \tau =$ constant. The DBING $D1$-brane action has been studied by the present authors (Kulshreshtha and Kulshreshtha, in press, 2003a) and for further details we refer the reader to Kulshreshtha and Kulshreshtha (in press, 2003a).

The problem of operator ordering occurring here while making a transition from EWST Dirac brackets to the corresponding EWST commutation relations can be resolved here as explained in Section 3, by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators (Gitman and Tyutin, 1990; Kulshreshtha and Kulshreshtha, 1993c, 2002a,b, 2003a,b, in press; Kulshreshtha *et al.*, 1993a, 1994a, 1993b, 1994b, 1993d,e; Maharana, 1983; Senjanovic, 1976).

It is important to mention here that in our work we have not imposed any boundary conditions (BC's) for the open and closed strings separately. There are two ways to take these BC's into account: (a) One way is to impose them directly in the usual way for the open and closed strings separately in an appropriate manner (Abou and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b, in press;

Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996), (b) an alternative second way (Chu and Ho, 2000; Sheikh-Jabbari and Shirzad, 1999) is to treat these BC's as the Dirac primary constraints (Chu and Ho, 2000; Sheikh-Jabbari and Shirzad, 1999) and study the theory accordingly (Chu and Ho, 2000; Sheikh-Jabbari and Shirzad, 1999). At present our related work is underway and would be reported later.

REFERENCES

Abou Zeid, M. and Hull, C. M. (1997). *Intrinsic Geometry of D-Branes*, hep-th/9704021.

Aganagic, M., Park, J., Popescu, C., and Schwarz, J. (1997). *Dual D-Brane Actions*, hep-th/9702133.

Brink, L. and Henneaux, M. (1988). *Principles of String Theory*, Plenum, New York.

Chu, C. S. and Ho, P. M. (2000). Constrained quantization of open string in background B field and noncommutative D-brane. *Nuclear Physics B* **568**, 447; arXiv: hep-th/ 9906192; (CITATION = HEP-TH 9906192).

de Alwis, S. P. and Sato, K. (1996) *Physical Review D* **53**, 7187–7196.

Dirac, P. A. M. (1950). *Canadian Journal of Mathematics* **2**, 129.

Gitman, D. M. and Tyutin, I. V. (1990). *Quantization of Fields with constraints*, Springer-Verlag, Berlin.

Johnson, C. V. (2000). *D-Brane Primer*, hep-th/0007170.

Kulshreshtha U. and Kulshreshtha, D. S. (2002a). *International Journal of Theoretical Physics* **41**, 1961–1976.

Kulshreshtha U. and Kulshreshtha, D. S. (2002b). *International Journal Theoretical Physics* **41**, 2411–2426.

Kulshreshtha U. and Kulshreshtha, D. S. (2003a). *European Physical Journal C* **29**, 453-461.

Kulshreshtha U. and Kulshreshtha, D. S. (2003b). *Physics Letters B* **555**(3–4), 255–263.

Kulshreshtha U. and Kulshreshtha, D. S. (2004). *International Journal of Theoretical Physics* **43**, 2355–2369.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993a). *Proceedings of the IInd Workshop on Constraint Theory and Quantization Methods*, Montepulciano (Siena), Italy, June 30–July 1 (Invited Talk by D. S. Kulshreshtha).

Kulshreshtha, Usha, Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993b). *Zeitschrift fur Physik C* **60**, 427–431.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993c). *Helvetica Physica Acta* **66**, 737–751.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993d). *Helvetica Physica Acta* **66**, 752–794.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993e). *Physical Review D* **47**, 4634–4638.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1994a). Proceedings of the IInd workshop on "Constraint Theory and Quantization Methods," Montepulciano (Siena), Italy, June 30-July 1, 1993. World Scientific, Singapore, pp. 305–327.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1994b). *Zeitschrift fur Physik C* **64**, 169–176.

Luest, D. and Theisen, S. (1989). *Lectures in String Theory: Vol. 346, Lecture Notes in Physics*, Springer-Verlag, Berlin.

Maharana, J. (1983). *Physics Letters B* **128**, 411.

Maharana, J. (2000). Recent Developements in String Theory. In *Quantum Field Theory*, A. N. Mitra, ed., pp. 611–664, hep-th/9911200.

Mukhi, S. (1997). *Recent Developements in String Theory: A Brief Review for Particle Physicists*, hep-ph/9710470 (TIFR/TH/97-55).
Schmidhuber, C. (1996) *Nuclear Physics B* **467**, 146–158.
Senjanovic, P. (1976). *Annals of Physics (N.Y.)* **100**, 227–261.
Sheikh-Jabbari, M. M. and Shirzad, A. (1999). *Boundary Conditions as Dirac Constraints*, hep-th/9907055.
Tseytlin, A. A. (1996). *Nuclear Physics B* **469**, 51–57.